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LETTER TO THE EDITOR

The half-periodic Josephson effect in an s-wave superconductor–normal-metal–d-wave superconductor junction

Alexandre M Zagoskin

Department of Physics and Astronomy, The University of British Columbia, 6224 Agricultural Road, Vancouver, B.C., V6T 1Z1, Canada†

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Abstract. We show that in a long clean s-wave superconductor–normal-metal–d-wave superconductor junction the Josephson current is periodic in the superconducting phase difference φ with period π instead of 2π. The frequency of the ac Josephson effect and the period of the magnetic interference pattern become $2\omega_J = 4eV$ and $\Phi_0/2$ respectively. This effect is due to the coexistence in the normal layer of current-carrying Andreev levels with phase differences φ and $\varphi + \pi$, and is robust with respect to finite temperature and weak elastic scattering.

The Josephson effect in superconductor–normal-metal–superconductor (SNS) junctions differs from the effect in tunnelling junctions in several important respects. In particular, in a long clean SNS junction at $T = 0$, the Josephson current, $I_J(\varphi)$, is a sawtooth function of the superconducting phase difference φ (Ishii 1970, Svidzinskii *et al* 1971), in contrast to the $\sin\varphi$ dependence in tunnelling junctions (Barone and Paternò (1982), ch 1). The reason for this behaviour is that the Josephson current is transferred through the normal layer by current-carrying Andreev levels, formed due to subgap Andreev reflections of electrons and holes from the non-diagonal pairing potential of superconductors (Kulik 1969, Bardeen and Johnson 1972). The positions of these levels, and therefore the current, depend on φ.

In this letter we consider the case in which one of the superconductors has d-wave pairing symmetry: an s-wave superconductor–normal-metal–d-wave superconductor (SND) junction (figure 1). There is a strong evidence that such pairing is realized in superconducting cuprates—in particular, results obtained from phase-sensitive measurements (Van Harlingen 1995, Tsuei *et al* 1996, Kouznetsov *et al* 1997). The experimental realization of an SND junction is feasible, and could provide additional information on the superconductivity in cuprates. Various types of Josephson junction between conventional and d-wave superconductors are being actively investigated (Yip 1993, Devereaux and Fulde 1993, Tanaka 1994, Zhu *et al* 1996, Zagoskin 1997, Riedel and Bagwell 1997, Huck *et al* 1997). In particular, Yip (1993) considered a point contact between superconductors with different pairing symmetries (a contact with length $L \ll \min(\xi_0)$, the lesser of the superconducting coherence lengths of the two superconductors). He demonstrated that the period of the $I_J(\varphi)$ dependence can be $2\pi/n$, with n an integer. This result could not be directly applied to the case of cuprates, where the opposite limit ($L \gg \max(\xi_0)$) is more likely to hold. Tanaka (1994) showed that in a tunnelling junction parallel to the c-axis the Josephson current is

† E-mail: zagoskin@physics.ubc.ca.

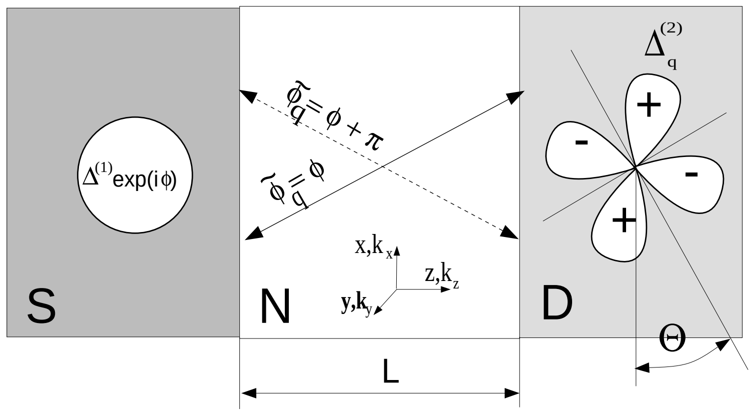

Figure 1. An s-wave superconductor–normal-metal–d-wave superconductor junction (an SND junction). We can choose a real order parameter for the d-wave superconductor. The Andreev zero- and π-levels are shown schematically.

governed by the contribution of the second order in Δ, leading to a $\sin 2\varphi$ current–phase dependence.

In this letter we show that SND junctions will exhibit a half-periodic Josephson effect ($I(\varphi)$ being a π-periodic function of the phase), because they contain two distinct sets of Andreev levels, coupled to positive and negative lobes of the d-wave order parameter, respectively.

The physical picture of the effect is as follows. In d-wave superconductors the order parameter can have either sign, according to the direction of the momentum on the Fermi surface, and so can be treated as an additional, intrinsic phase π (Sigrist and Ueda 1991). Therefore, the Andreev levels in the SND junction depend (instead of on φ) on the effective phase difference:

$$\tilde{\varphi}^{(\boldsymbol{k}_\parallel)} = \varphi + \pi\vartheta(-\Delta_{\hat{\boldsymbol{q}}}) \tag{1}$$

where $\vartheta(x)$ is the Heaviside step function. The unit vector $\hat{\boldsymbol{q}}$ gives the direction of the wave vector of the transmitted state in a d-wave superconductor, $\boldsymbol{q}$, with components $\boldsymbol{k}_\parallel$ (parallel to the interface) and $q_z(E)$ (see figure 1). There are thus two sets of Andreev levels in the normal layer: with $\tilde{\varphi} = \varphi$ ($\Delta_{\hat{\boldsymbol{q}}}$ positive) and with $\tilde{\varphi} = \varphi + \pi$ ($\Delta_{\hat{\boldsymbol{q}}}$ negative). The Josephson current through the π-levels will be a 2π-periodic function of the phase, shifted by π with respect to the current carried by the zero-levels, like in the so-called π-junction with a negative critical current (Sigrist and Rice 1992).

The resulting current is π-periodic in phase (see figure 2(b)). From the Josephson relation $\partial_t\varphi = 2eV$ we see that the Josephson frequency doubles: $\tilde{\omega}_J = 2\omega_J = 4eV$.

In our further analysis we assume that the following conditions are satisfied:

$$\max(\xi_0) \ll L \ll l_T, l_\varepsilon, l_i \tag{2}$$

where L is the width of the normal-metal layer, $\max(\xi_0)$ is the larger of the correlation lengths of the two superconductors, l_i, l_ε are the elastic and inelastic scattering length respectively, and $l_T = v_F/(2\pi k_B T)$ is the normal-metal coherence length in the clean limit. The latter is the characteristic coupling length in the SNS junction (Kulik 1969), and can be much larger than ξ_0 (especially in the case of cuprates). In silver, e.g. at 1 K, $l_T = 1.67 \times 10^{-4}$ cm, compared to $\xi_0 \sim 10^{-5}$ cm typical for conventional superconductors.

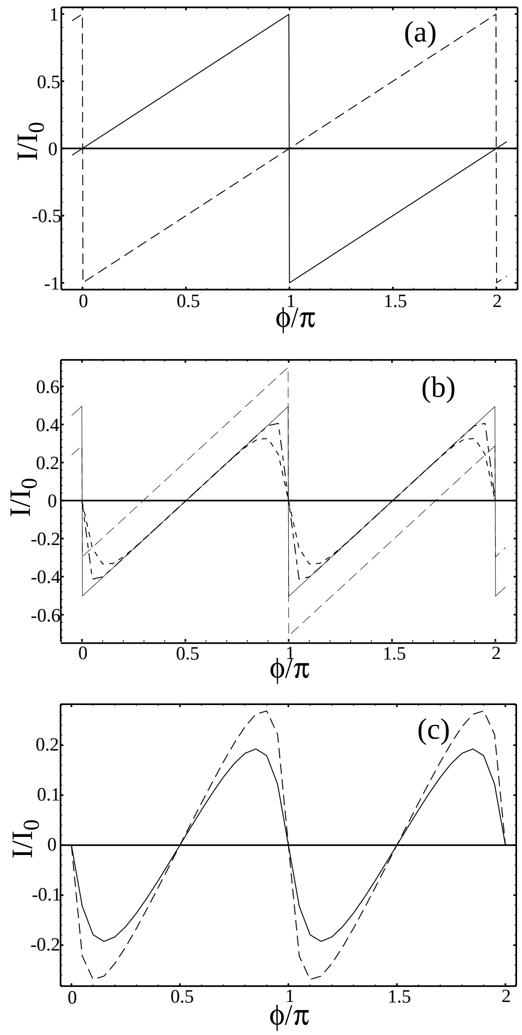

Figure 2. (a) Current–phase characteristics of a clean SNS junction at zero temperature: the conventional ('zero') junction (solid line); and the π-junction (dashed line). (b) Current–phase characteristics of the half-periodic Josephson current: $Z = 0, T = 0$ (solid line); $Z = 2^{1/2} - 1$, $T = 0$ (dashed line); and $Z = 0$, $L/l_T = 0.2$ (chain line); and $Z = 0$, $L/l_T = 0.5$ (dotted line). The imbalance function Z is defined in equation (9). (c) Current–phase characteristics of the half-periodic Josephson current at finite normal scattering: $Z = 0$; $L/l_i = 0.05$ (dashed line); and $L/l_i = 0.1$ (solid line).

Andreev levels in the normal layer are obtained by solving Bogoliubov–de Gennes equations for the two-component wave function in both of the superconductors and the normal layer, separately for each $\boldsymbol{k}_\parallel$-mode, and matching the wave functions at the interfaces (see e.g. Hurd and Wendin (1994), and references therein). For the SND junction we have

$$\begin{pmatrix} \mathcal{H} - (k_F^2 - k_\parallel^2)/2m & \Delta(z) \\ \Delta^*(z) & -(\mathcal{H} - (k_F^2 - k_\parallel^2)/2m) \end{pmatrix} \begin{pmatrix} u(z) \\ v(z) \end{pmatrix} = E \begin{pmatrix} u(z) \\ v(z) \end{pmatrix}. \tag{3}$$

Here $\mathcal{H} = -(1/2m)\nabla_z^2$ is the one-particle Hamiltonian; $\Delta(z)$ is the non-diagonal potential (which is zero in a normal layer, $\Delta^{(1)}\mathrm{e}^{\mathrm{i}\varphi}$ in an s-wave superconductor, and $\Delta^{(2)}_{\hat{q}}$ in a d-wave superconductor; we choose real $\Delta^{(1)}$, $\Delta^{(2)}_{\hat{q}}$). This standard approximation is justified by the condition $\xi_0 \ll L$. We assume for simplicity the same value of k_F in all three regions. The corrections due to the differences in k_F, and the finite normal scattering by the interface will be considered elsewhere.

In the normal layer the normal component of the momentum of an electron (hole) with energy E and tangential momentum $\boldsymbol{k}_\parallel$ is

$$k_z^{e,h}(E) = \sqrt{k_F^2 - k_\parallel^2 \pm 2mE}.$$

In the superconductor, for a subgap quasiparticle, $E < |\Delta|$, this transforms into

$$q_z^{\pm}(E) = k_z(0)\sqrt{1 \pm \mathrm{i}\frac{|\Delta|}{k_z(0)^2/2m}\sqrt{1-\frac{E^2}{|\Delta|^2}}}. \tag{4}$$

If $|\Delta| \ll k_z^2(0)/2m$, then $\mathrm{Re}\, q_z^{\pm} \approx k_z(0)$, i.e. the quasiparticle momentum does not change direction in the superconductor: $\hat{\boldsymbol{q}} \approx \hat{\boldsymbol{k}}$, $\boldsymbol{k} = (\boldsymbol{k}_\parallel, k_z)$. Therefore the momentum of the quasiparticle in the normal layer determines the value $\Delta_{\hat{\boldsymbol{k}}}$ of the d-wave order parameter entering the Bogoliubov–de Gennes equations (3), and the effective phase difference for a given Andreev level (see figure 1).

Under the above conditions, the energies of low-lying Andreev levels ($E \ll |\Delta|$) are given by

$$(k_z^e(E) - k_z^h(E))L \pm \tilde{\varphi}^{(\boldsymbol{k}_\parallel)} = \pi(2n+1) \qquad n = 0, \pm 1, \pm 2, \ldots. \tag{5}$$

This is a direct generalization of the result given by Kulik (1969).

The Josephson current through the contact can be calculated using the low-energy excitation spectrum determined from (5) (Bardeen and Johnson 1972, van Wees *et al* 1991). At $T = 0$,

$$I_J(\varphi) = \sum_{\boldsymbol{\kappa}} \frac{2ev_{Fz}^{(\boldsymbol{\kappa})}}{\pi L}\sum_{n=1}^{\infty}(-1)^{n+1}\frac{\sin n\tilde{\varphi}^{(\boldsymbol{\kappa})}}{n}. \tag{6}$$

Here $\tilde{\varphi}^{(\boldsymbol{\kappa})}$ is the effective phase shift, and $v_{Fz}^{(\boldsymbol{\kappa})}$ is the component of the Fermi velocity that is normal to the interface in a mode with tangential momentum $\boldsymbol{k}_\parallel = \boldsymbol{\kappa}$: $(mv_{Fz}^{(\boldsymbol{\kappa})})^2 + \boldsymbol{\kappa}^2 = k_F^2$. The summation

$$\sum_{\boldsymbol{\kappa}} = S\int_{|\boldsymbol{\kappa}|\leqslant k_F} \mathrm{d}\boldsymbol{\kappa}_x\ \mathrm{d}\boldsymbol{\kappa}_y$$

is extended over all allowed tangential modes†; S is the area of the junction.

It follows from (6) that the Josephson current in a clean SND junction is indeed a sum of independent contributions of zero- and π-modes:

$$I_J(\varphi) = I_0(\alpha^{(+)}F(\varphi) + \alpha^{(-)}F(\varphi+\pi)) = \frac{I_0}{2}(F(\varphi) + F(\varphi+\pi) + Z(F(\varphi) - F(\varphi+\pi))). \tag{7}$$

Here

$$F(x) = \frac{2}{\pi}\sum_{n=1}^{\infty}(-1)^{n+1}\frac{\sin nx}{n}$$

† The corrections from 'grazing' trajectories (with $k_\parallel \approx k_F$) are negligible, since the parameter $|\Delta^{(1,2)}|/E_F \ll 1$.

is the 2π-periodic sawtooth of unit amplitude; also

$$I_0 = \frac{e}{L}\sum_{\kappa} v_{Fz}^{(\kappa)}$$

is the critical current of a clean SNS junction at zero temperature (Svidzinskii *et al* 1971, Bardeen and Johnson 1972); and the weight factors are

$$\alpha^{(\pm)} = {\sum_{\kappa}}^{\pm} v_{Fz}^{(\kappa)} \Big/ \sum_{\kappa} v_{Fz}^{(\kappa)}$$

where the $\pm$-summation is extended over zero- and π-modes respectively.

The ratio of the two contributions in (7) generally depends on the orientation of the d-wave superconductor with respect to the interface, parametrized by certain angles θ_a, through the imbalance function

$$Z(\theta_a) = \frac{\alpha^{(+)}(\theta_a) - \alpha^{(-)}(\theta_a)}{\alpha^{(+)}(\theta_a) + \alpha^{(-)}(\theta_a)} \qquad |Z(\theta_a)| \leqslant 1. \tag{8}$$

$Z = \pm 1$ corresponds to purely zero- or π-junction, respectively; $Z = 0$ when the amplitudes of the contributions from zero- and π-levels are equal. Due to the non-sinusoidal form of the current–phase characteristics, the current is not zero if $Z(\theta_a) = 0$. On the contrary, the resulting current is π-periodic in phase (see figure 2(b)), and its critical value is halved. Generally, the ground state of the system is doubly degenerate, with the equilibrium phase difference across the junction

$$\pm\phi_0 = \pm\frac{1-Z}{2}\pi.$$

The simplest case is an SND junction where the x^2–y^2-plane of the d-wave superconductor is parallel to the interface (see figure 3(a)); then $Z(\theta_a) \equiv 0$ by symmetry. For cuprates, this (001) plane is also the easiest-cleavage plane, which makes it the best candidate for experimental observation of the effect.

Another possibility is to choose the interface normal to that plane. In this case the imbalance function depends on a single angle θ between the SN interface and the nodal direction in the x^2–y^2-plane (see figure 3(b)), and $Z(\theta) = \pm(\sqrt{2}\cos[\theta - \pi/4] - 1)$ $(0 \leqslant \theta \leqslant \pi/2)$. Since $|Z(\theta)| \leqslant \sqrt{2} - 1$, the Josephson current in this case always contains a π-periodic component (see figure 2(b))†.

Starting from (7), it is straightforward to calculate the magnetic interference pattern (the dependence of the critical current on the external magnetic field) in the junction. Using the standard approach (Barone and Paternò (1982), ch 4), we find

$$I_c(\nu) = \frac{1}{2\pi\nu} \max_{-\pi\leqslant\vartheta\leqslant\pi} \int_0^{2\pi\nu} \mathrm{d}\phi \; I_J(\phi + \vartheta) \tag{9}$$

and finally

$$\frac{I_c(\nu)}{I_0} = \begin{cases} \dfrac{\{\nu\}}{2|\nu|}(1 + |Z| - 2\{\nu\}) & (0 \leqslant \{\nu\} < 1/2) \\ \dfrac{1-\{\nu\}}{2|\nu|}(-1 + |Z| + 2\{\nu\}) & (1/2 \leqslant \{\nu\} < 1) \end{cases} \tag{10}$$

† It is worth noting that in the case of a DND junction between two d-wave superconductors, the half-periodic Josephson effect is present as well, but it disappears at certain orientations of the crystals. For example, if the junction interface is parallel to the x^2–y^2-plane of both superconductors, then $Z(\alpha) = \pm(1 - 4\alpha/\pi)$. Here $0 \leqslant \alpha \leqslant \pi/2$ is the angle between the nodal directions in the superconductors, and the effect is absent if $\alpha = 0$ or $\pi/2$.

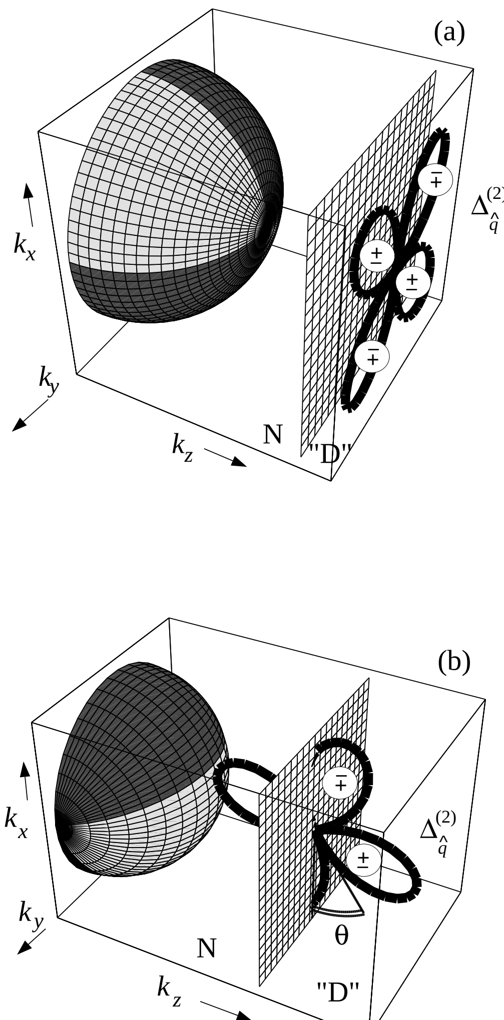

Figure 3. The systems used for the calculation of the imbalance function for different orientations of the d-wave superconductor with respect to the interface: white regions on the Fermi surface (in the normal metal) correspond to zero-modes, and shadowed regions correspond to π-modes, or vice versa. (a) The interface parallel to the x^2–y^2-plane. (b) The interface normal to the x^2–y^2-plane; θ is the angle between the SN interface and the nodal direction in the x^2–y^2-plane.

where $0 \leqslant \{x\} < 1$ is the fractional part of x (figure 4). The interference pattern given by (10) is remarkable in that, while preserving the trademark triangular central peak of an SNS junction, it presents a continuous transition from the standard period Φ_0 ($|Z| = 1$) to $\Phi_0/2$ ($|Z| = 0$). This behaviour is clearly distinct from the pattern in a 0–π tunnelling junction (a combination of a conventional and a π-junction in parallel), where the dependence $I_c(\Phi) \sim \sin^2(\pi\Phi/2\Phi_0)/|\pi\Phi/2\Phi_0|$ was predicted (Kirtley *et al* 1997) and recently reported (Kouznetsov *et al* 1997).

So far we have not considered the effects of non-magnetic impurity scattering and finite temperature. At finite temperatures, the condition $E \ll |\Delta^{(i)}|$ will be obeyed for almost all

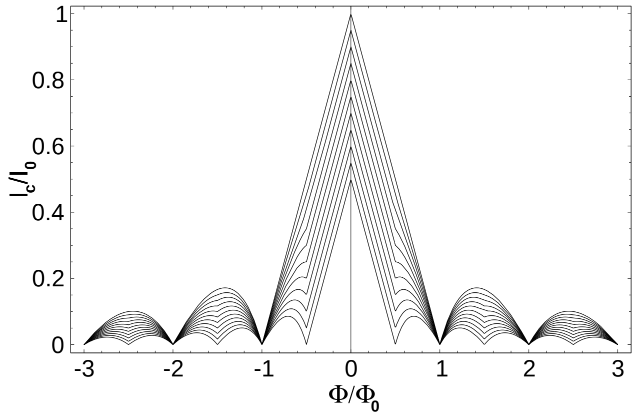

Figure 4. The magnetic interference pattern in an SND junction. The dependence of the critical current density on Φ/Φ_0 is given for $|Z| = 1, 0.9, \ldots, 0$ (top to bottom).

of the Fermi surface if

$$k_B T \sim E \ll |\Delta^{(1)}|; \max|\Delta^{(2)}_{\hat{k}}|. \tag{11}$$

Corrections to the current from the regions close to the nodal lines will be small, as they are governed by the same parameter, $k_B T/\max|\Delta^{(2)}_{\hat{k}}|$. This additional condition on the temperature is in fact less restrictive than the one following from (2).

Weak elastic scattering can be taken into account via the broadening of Andreev levels due to the finite scattering time, $\tau = l_i/v_F$ (Kulik and Mitsai 1975). Using the approach of Kadigrobov *et al* (1995), we find for $L/l_i, L/l_T \ll 1$

$$I_J(\varphi) = \sum_{\kappa} \frac{2ev^{(\kappa)}_{Fz}}{\pi L} \sum_{n=1}^{\infty} (-1)^{n+1} \exp\left[-2n\frac{L}{l^{(\kappa)}_i}\right] \frac{(L/l^{(\kappa)}_T) \sin n\tilde{\varphi}^{(\kappa)}}{\sinh(nL/l^{(\kappa)}_T)} \tag{12}$$

(see figures 2(b), 2(c)). Here $l^{(\kappa)}_i = l_i v^{(\kappa)}_{Fz}/v_F$ and $l^{(\kappa)}_T = l_T v^{(\kappa)}_{Fz}/v_F$ are the effective scattering and coherence lengths in mode $\boldsymbol{\kappa}$, respectively. The effect thus survives finite temperature and weak elastic scattering in the normal layer.

In the limit of strong elastic scattering, $L \gg l_i$, or at higher temperatures, $L \gg l_T$, the sawtooth current–phase dependence in long SNS junctions reduces to a sinusoidal one (Kulik and Mitsai 1975), and the effect disappears due to the cancellation of the contributions from the zero- and π-modes.

In conclusion, we have demonstrated that in a long clean SND junction two groups of Andreev levels coexist, coupled to positive and negative lobes of the d-wave order parameter, and thus with effective phases shifted by π. (The relative weights of these groups depend on the orientation of the d-wave superconductor with respect to the NS interface.) As a result, the period of the Josephson current is halved, and the ac Josephson frequency doubles. The magnetic interference pattern retains its triangular central peak, but becomes periodic with period $\Phi_0/2$ as a function of the external magnetic flux. The effect is robust with respect to finite temperature and weak elastic scattering in the normal layer, and provides a new tool for use in the investigation of pairing symmetry in unconventional superconductors.

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